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Fluid motions due to an electric current source

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A concentrated electric current enters a region of fluid through an interface. It is shown that the magnetic field due to the current in general gives rise to rotational magnetic forces which must cause motions of the fluid. In particular the paper solves the axisymmetric non-linear problem in which a concentrated current enters a semi-infinite region of inviscid conducting fluid of constant density, inducing an inwards flow along the wall and a jet away from the wall opposite the current source. The case treated is the practically realistic one in which the effective magnetic Reynolds number is small and the current flows isotropically from the source. The first-order perturbation of this current distribution by electromagnetic induction is also calculated.

An analytical solution is possible because the non-linear equation of motion happens to be a *linear* equation in the square of the Stokes stream function. The motion is analytically related to viscous jet flows discussed by Slezkin, Landau and Squire.

1. Introduction

Situations where a large electric current enters a body of conducting liquid or gas at a more-or-less concentrated point located on a plane boundary of the fluid are fairly common in electrotechnology. Sometimes the current is passing between a liquid and a gas, as in arc welding, arc furnaces or mercury arc rectifiers. It has been recognized that the magnetic forces due to the magnetic field of the current can give rise to violent motions in the fluid or both fluids (see, for instance, Maecker 1955, Zhigulev 1960 and Amson 1965) but no adequate fluid mechanical investigation of the problem appears to have been undertaken. Apart from Zhigulev's short note, theory hitherto has taken a primitive view of the mechanics, usually in terms of 'magnetic pressure', without adequate recognition that fluid pressure also acts and that therefore it is the *rotationality* of the magnetic force that determines the motion, at least if compressibility is unimportant and the fluid has no free surfaces and uniform density. When the surface is unconstrained, the interplay between magnetic forces and fluid pressure in the boundary condition makes the problem more complicated. As a first step towards understanding these important flows, we confine attention here to constant density flows with fixed boundaries and an axis of symmetry.

The electric current is supplied through the wall where $\theta = \frac{1}{2}\pi$ (if spherical polar co-ordinates r, θ are used as in figure 1). At least in the vicinity of the fluid, the current must flow in an axisymmetric manner behind the wall so as to preserve the axisymmetry. Then the magnetic field **B** is purely azimuthal (of magnitude B) and it is easily verified that curl $\mathbf{j} \times \mathbf{B}$ is an azimuthal vector of magnitude $2Bj_s/s$, where s is distance from the axis, **j** is the current density and j_s its component normal to the axis. The direction of curl $\mathbf{j} \times \mathbf{B}$ is indicated by the circles bearing arrows in figure 1, and this holds whatever the direction of **j**, since **B** reverses if **j** does. By the same token, the effect occurs also with alternating current, unless modified by 'skin effect'. Thus, wherever current lines diverge, vorticity is generated and one conjectures that motions such as those indicated by the dashed lines in figure 1 will occur. Zhigulev (1960) pointed out that the fluid could not remain at rest.

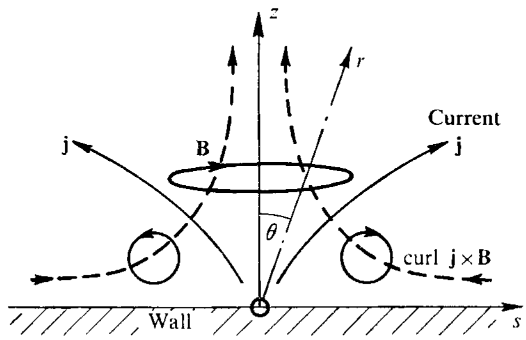

FIGURE 1

An important point is that this phenomenon is essentially three-dimensional; the two-dimensional analogue does not exist, as Zhigulev remarked. Then the magnetic forces are impotent because

$$\text{curl }\mathbf{j}\times\mathbf{B} = 0, \quad \text{for} \quad \mathbf{j}\times\mathbf{B} = (\mathbf{B}\cdot\text{grad})\mathbf{B}/\mu - \text{grad }\mathbf{B}^2/2\mu \quad \text{and} \quad \mathbf{B}\cdot\text{grad} = 0$$

in the two-dimensional case.

Many interesting equations immediately arise. Suppose, for instance, that the fluid is confined in a closed, axisymmetric container, within which it eddies round under the action of $\mathbf{j} \times \mathbf{B}$ forces, the flow being in meridional planes unless some instability sets in. It would be possible to arrange for the current to be spreading everywhere with j_s of constant sign, e.g. by the use of one large and one small electrode, so that vorticity generation is everywhere of the same sign. The circulating fluid would thus accelerate and could not reach a steady state unless some new mechanism intervened. Note that this acceleration would always happen, however feeble the current supply, provided one waited long enough. One possibility is that ultimately viscosity (or turbulence) would provide a brake, with boundary layers if the viscosity were relatively weak. Note, however, that the fluid would *all* have to enter the boundary layers somewhere to lose the vorticity which it had gained from curl $\mathbf{j} \times \mathbf{B}$ throughout the fluid. A more subtle limiting mechanism, which might occur for other values of the prevailing parameters, would be that the motion could induce e.m.f.s which strongly modified the current distribution so that curl $\mathbf{j} \times \mathbf{B}$ vanished over large regions of the flow or even reversed locally, so that the fluid could settle down in a steady state of alternately gaining and losing vorticity. These remarks hint at the existence of a whole range of apparently formidable non-linear problems. In the welding application the acceleration process is terminated by the intermittent transfer of liquid metal drops from the electrode to the workpiece.

This paper treats what must surely be the simplest configuration of this general kind. The current enters the fluid at a point (the origin in figure 1). It is now assumed that the container is so large that the fluid can be regarded as coming from a region (θ near $\frac{1}{2}\pi$) where the fluid is virtually at rest and devoid of vorticity; also that the current is able to take whatever route is demanded by the problem to remote electrodes at large r. The vorticity is continually being imparted to fresh fluid and so a steady state can reasonably be sought. If the vorticity were rising, the general flow would be accelerating, so reducing the transit-time of fluid particles through the most active region near the origin until stable dynamic equilibrium was reached. The fact that the vorticity lines, which are azimuthal circles, are continually being shortened by the in-flow also should have a 'subduing' effect on the flow. It should not be necessary to invoke viscosity to achieve a steady state, and we shall therefore neglect it, for simplicity and to render the problem more tractable mathematically.

2. Formulation of the problem

It is convenient to express all quantities in terms of B, the magnetic field, and a Stokes stream function ψ. Note that Bs/μ acts like a Stokes stream function with respect to current flow because the equation curl $\mathbf{B} = \mu\mathbf{j}$ implies that

$$\mu s j_s = -\partial(Bs)/\partial z, \quad \mu s j_z = \partial(Bs)/\partial s, \tag{1}$$

if z is the axial cylindrical polar co-ordinate (see figure 1). The fluid is assumed non-magnetic so that μ takes its vacuum value. The components of velocity are given by

$$s v_s = -\partial\psi/\partial z, \quad s v_z = \partial\psi/\partial s. \tag{2}$$

The governing equations of steady inviscid MHD, after elimination of the electric field and the pressure, become

$$\mu\sigma \,\mathrm{curl}\, \mathbf{v}\times\mathbf{B} = \mathrm{curl}\,\mathrm{curl}\,\mathbf{B} \tag{3}$$

and

$$\mu\rho\,\mathrm{curl}\,\boldsymbol{\omega}\times\mathbf{v} = \mathrm{curl}\,(\mathrm{curl}\,\mathbf{B})\times\mathbf{B}, \tag{4}$$

if σ = conductivity (assumed uniform), ρ = density and $\boldsymbol{\omega}$ = curl $\mathbf{v}$, the vorticity. The boundary conditions are that $\psi = 0$, say, at $\theta = 0$ and $\frac{1}{2}\pi$, that the vorticity tends to zero at large s, i.e. as $\theta \to \frac{1}{2}\pi$, that $B = 0$ at $\theta = 0$ (since a finite current filament along the axis could not persist in a finite conductor) and that

$$Br = Bs = \mu J/2\pi \quad \text{at} \quad \theta = \tfrac{1}{2}\pi, \tag{5}$$

J being the total current supplied through the origin to the fluid. Clearly v_s must tend to zero on the axis for there are no fluid sources or sinks there, but singular behaviour at the origin may be expected because of the infinite current density there.

The problem is characterized by only three assignable magnitudes, namely $\mu\sigma$, $\mu\rho$ and μJ; there is no length or velocity scale. There is, nevertheless, a characteristic dimensionless parameter, $K = \mu J \mu\sigma/(\mu\rho)^{\frac{1}{2}}$ or $\mu^{\frac{3}{2}} J\sigma/\rho^{\frac{1}{2}}$, which must govern the form of the solutions. The case of large K must be that where inertia is relatively unimportant and the fluid moves so nimbly that it convects the current flow into a form where curl $\mathbf{j} \times \mathbf{B} \approx 0$ over most of the flow field in the absence of significant inertial resistance. This is the case discussed incompletely by Zhigulev (1960).

The case of small K, on the other hand, must be the case where the sluggish fluid induces negligible e.m.f.s and the current therefore spreads isotropically outwards from the origin, thereby determining the curl $\mathbf{j} \times \mathbf{B}$ field which drives the fluid motion. In other words, K is a form of magnetic Reynolds number R_m; we shall find that, at least at low K, the velocities are of order $(J/r)(\mu/\rho)^{\frac{1}{2}}$, so that R_m ($= \mu\sigma \times$ velocity $\times r$) becomes $\mu\sigma \times J(\mu/\rho)^{\frac{1}{2}}$, i.e. K. Note that R_m, so defined, is the same throughout the field of flow and we do not have high R_m behaviour at large distances, low R_m behaviour near the origin, as occurs in some problems.

If we insert typical, reasonable magnitudes, whether for a liquid metal or an ionised gas, we find that K is usually very small compared with unity. It could only become large with hot, tenuous plasma having high σ and low ρ, or in very high power devices involving liquid metals. In this paper we therefore concentrate on the low K case where the magnetic field may be easily found first, and then the non-linear fluid mechanics solved subsequently.

Further inferences may be extracted from the dimensional arguments. Both B and ψ are functions of r, θ, $\mu\sigma$, $\mu\rho$ and μJ. It proves more convenient to use $c = \cos\theta = z/r$ instead of θ as a variable. It follows that the functions are such that

$$Bs/\mu J = f(c, K) \tag{6}$$

and

$$\psi\rho^{\frac{1}{2}}/rJ\mu^{\frac{1}{2}} = g(c, K), \tag{7}$$

for dimensional consistency. Note the tactical choice of Bs but ψ/r in (6) and (7). These equations indicate that the problem is self-modelling, and Bs/μ and ψ/r are constant along rays of given inclination through the origin. Consequently the current always flows along these rays, whatever the value of K. The radial current density j_r is given by

$$j_r = -J\dot{f}/r^2, \tag{8}$$

where a dot denotes differentiation with respect to c. Equation (7) is reminiscent of the self-similar, viscous, non-conducting flows solved by Slezkin, Landau and Squire and reviewed by Whitham (1963, pp. 150–155). The integrability of the equations in those cases gives us encouragement here.

From (6) and (7), the azimuthal component of (3) becomes

$$\ddot{f}(1-c^2)^2 = K\{2cfg + (1-c^2)(2f\dot{g} + \dot{f}g)\}. \tag{9}$$

The vorticity ω (in the azimuthal direction) is given by

$$\omega = -(\mu^{\frac{1}{2}}J/\rho^{\frac{1}{2}}r^2)(1-c^2)^{\frac{1}{2}}\ddot{g}, \tag{10}$$

a remarkably simple expression, which does not appear to have been referred to in connexion with the earlier work on the viscous flows governed by relations like (7). From (10), the azimuthal component of (4) becomes

$$2(1-c^2)(g\dddot{g}+3\dot{g}\ddot{g}) = (1-c^2)(d/dc)^3g^2 = 4f\dot{f}. \tag{11}$$

Thus the non-linear equation of motion is in fact a rather simple linear differential equation in g^2.

3. Non-conducting flows

In the absence of magnetic forces, equation (11) with $f = 0$ yields a family of ordinary, non-linear, axisymmetric, inviscid rotational flows. The solutions are of the form

$$(\psi/r)^2 \propto g^2 = Ac^2+Bc+C, \tag{12}$$

where A, B and C are constants. They should be compared with the corresponding family of potential flows derived from (9) with ω set equal to zero, so that

$$g = Dc+E, \tag{13}$$

with D and E constant. These solutions are those members of the family (12) for which $B^2 = 4AC$. They represent flows for which the streamlines are conics with the origin as focus and with their major axes in the z direction.

4. Conducting flows with $K \ll 1$

This is the case where the current and its magnetic field are expected to be undisturbed by the motion, and the current flows isotropically outwards from the origin. This is confirmed when K is set equal to zero in (9), which then becomes equivalent to the statement curl $\mathbf{j} = 0$. Now $\dot{f}$ = const. and (8) indicates that j_r is independent of θ, as expected. From the boundary conditions (5) it follows that

$$f = (1-c)/2\pi. \tag{14}$$

The remaining task is to solve (11) which may now be written

$$\left(\frac{d}{dc}\right)^3 g^2 = -\frac{1}{\pi^2(1+c)}. \tag{15}$$

This equation may be integrated three times to yield

$$g^2 = \{Ac^2+Bc+C-\tfrac{1}{2}(1+c)^2\log(1+c)\}/\pi^2, \tag{16}$$

and C must vanish because of the boundary condition $g = 0$ when $c = 0$.

We next consider the approaching flow at large r, finite z and small c. With arbitrary values of B, $\ddot{g} \propto c^{-\frac{3}{2}}$ as $c \to 0$ and, from (10), $\omega \propto r^{-\frac{1}{2}}$ at constant z as $r \to \infty$, as it does also for those rotational flows (12) for which $g = 0$ when $c = 0$. As $r \to \infty$ *along a streamline*, ω diverges without limit, however. In our problem we want the flow at large r and finite z to approach an irrotational flow, with ω falling to zero faster than $r^{-\frac{1}{2}}$ at large r and constant z. One choice of B permits this, namely, $B = \frac{1}{2}$, so as to cancel the c term from the logarithm expression in (16). Then $\ddot{g} \to$ const. as $c \to 0$, and $\omega \propto r^{-2}$ at constant z as $r \to \infty$. Moreover it will emerge later that the streamlines go to ∞ at constant z. This choice evidently corresponds to the case of the fluid irrotational at infinity. With $B = \frac{1}{2}$, (16) gives $g \approx (A-\frac{3}{4})^{\frac{1}{2}}c$, when c is small, which corresponds to that potential flow (13) for which $g = 0$ when $c = 0$.

The constant A is determined by the condition that ψ and hence $g = 0$ when $c = 1$(r finite). Hence $A = 2\log 2 - \frac{1}{2}$, and our solution becomes

$$g^2 = \{(4\log 2 - 1)c^2 + c - (1+c)^2 \log(1+c)\}/2\pi^2. \tag{17}$$

One curiosity of the solution is that the sign of g and the direction of flow are arbitrary. The flow *from* the region of zero vorticity is the natural one to choose; this is the one for which g is positive since then v_s is negative. In principle the reverse flow is possible, but it would involve the artificiality of fluid approaching with a very special distribution of vorticity which was then *just* destroyed by curl $\mathbf{j} \times \mathbf{B}$.

When c is small, the result (17) may be written

$$g^2 = \frac{c^2}{2\pi^2}\left\{4\log 2 - \frac{5}{2} - \frac{c}{3} + \frac{c^2}{12}\cdots\right\} \tag{18}$$

or, approximately, $g \approx 0{\cdot}117c$. Hence

$$\psi \simeq 0{\cdot}117(\mu^{\frac{1}{2}}J/\rho^{\frac{1}{2}})z, \tag{19}$$

which represents the approaching potential flow, with streamlines parallel to the wall with z constant. In calculating values of g, (17) is ill-conditioned and (18) is useful at small values of c. Similarly, when c approaches 1, it is useful to express g in terms of $\epsilon = 1 - c$, as follows:

$$\frac{\pi g}{(1-c^2)^{\frac{1}{2}}} = (\tfrac{3}{4} - \log 2)^{\frac{1}{2}}\left\{1 - \frac{\epsilon(1-\log 2)}{8(\frac{3}{4}-\log 2)}\cdots\right\}, \tag{20}$$

because $\psi\rho^{\frac{1}{2}}/sJ\mu^{\frac{1}{2}} = g/(1-c^2)^{\frac{1}{2}}$. Near $c = 1$, $\epsilon \approx \frac{1}{2}s^2/z^2$ and we may deduce that

$$\psi \approx 0{\cdot}076(\mu^{\frac{1}{2}}J/\rho^{\frac{1}{2}})s, \tag{21}$$

$$v_s \approx -(J/\pi)(\mu/\rho)^{\frac{1}{2}}(1-\log 2)s^2/8(\tfrac{3}{4}-\log 2)^{\frac{1}{2}}z^3,$$

and $$v_z \approx (J/\pi)(\mu/\rho)^{\frac{1}{2}}(\tfrac{3}{4}-\log 2)^{\frac{1}{2}}/s.$$

From (21) we see that the streamlines become straight and parallel to the axis at small θ. Note that $v_z \propto 1/s$, indicating a strong jet away from the wall with a singularity on the axis. This relatively weak singularity as $s \to 0$ arises because the streamline $\psi = 0$ passes through a region of infinite curl $\mathbf{j} \times \mathbf{B}$ at the origin. But v_s is not singular and there are no sources and sinks on the axis.

Figure 2 shows some typical streamlines. The asymptotes of the streamline $\psi =$ const. are

$$z \to 8{\cdot}55(\psi/J)(\rho/\mu)^{\frac{1}{2}} \quad \text{as} \quad s \to \infty$$

and $$s \to 13{\cdot}2(\psi/J)(\rho/\mu)^{\frac{1}{2}} \quad \text{as} \quad z \to \infty.$$

To establish practical orders of magnitude for the velocities, we may take, as typical values, $J = 100A$, $\rho = 10^3$ Kg/m^3 (a light liquid metal) and $s = 1$ cm $= 10^{-2}$ m. Then v_z at large z equals $2{\cdot}7$ cm/sec with this relatively modest current level. Note, however, that in a finite system, the fluid could recirculate repeatedly through the active region and attain very much higher velocities. In a gas the density would be much lower and the velocity higher.

Figure 2 reveals how the streamlines change rather abruptly from the converging, near-potential flow to the parallel, highly rotational, jet flow. In the former the fluid is accelerating because of the convergence, passing rapidly through the increasingly rotational $\mathbf{j} \times \mathbf{B}$ force field. This, combined with the

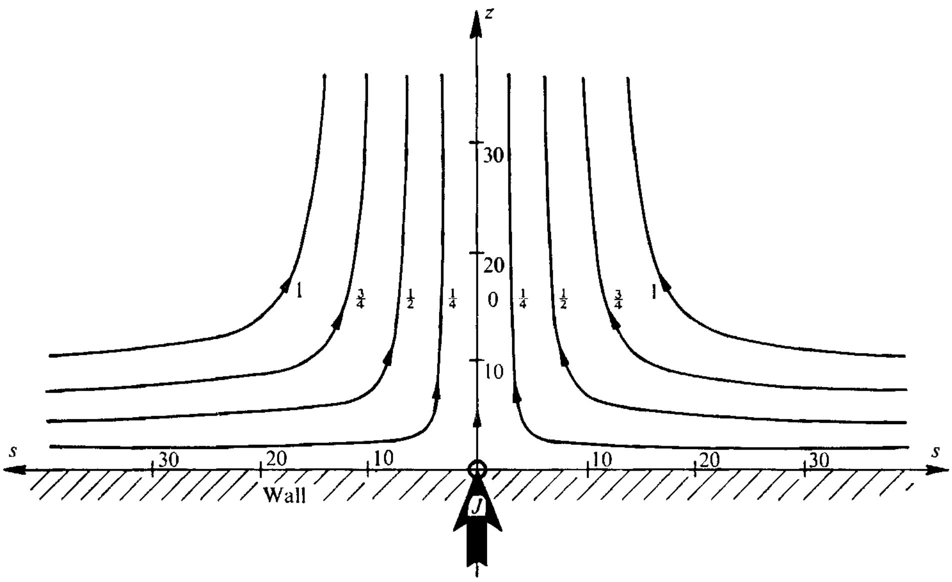

FIGURE 2. Typical streamlines at low K. The numbers by the curves are the values of $(\psi/J)(\rho/\mu)^{\frac{1}{2}}$, measured in the same length units as s and z.

progressive shortening of vorticity lines, keeps the vorticity at low levels until suddenly the régime switches rapidly, with a swift rise in vorticity and curvature of the streamlines, and the fluid escapes into the region of weaker $\mathbf{j} \times \mathbf{B}$ forces at large z, preserving its vorticity, as the vorticity-line-shortening ceases. The vorticity history of particles traversing the streamline $\psi = \text{const.}$ is given by the equation

$$\omega = -\frac{J^3}{\psi^2}\left(\frac{\mu}{\rho}\right)^{\frac{3}{2}}(1-c^2)^{\frac{1}{2}}g^2\ddot{g},$$

as c and g vary.

Some interest attaches to the pressure distribution along the wall at $z = 0$. As a first step towards this we need the value of v_s at the wall, which is

$$v_s = -\frac{J}{s\pi}\left(\frac{\mu}{\rho}\right)^{\frac{1}{2}}(2\log 2 - \tfrac{5}{4})^{\frac{1}{2}},$$

from (18). On the streamline at the wall, the $\mathbf{j} \times \mathbf{B}$ force is normal since $\mathbf{j}$ is along the streamline, and so Bernoulli's equation can be used for deducing the pressure p at the wall. The distribution is

$$p = p_0 - ks^{-2}, \tag{22}$$

where $k = (\mu J^2/\pi^2)(\log 2 - \frac{5}{8})$ and p_0 is the pressure on the wall at large s.

In a real situation the singularity of pressure at $s = 0$ would be removed by the finite size of a real electrode, or by viscous effects, or (in a gas) by compressibility effects. In a liquid the low pressures might conceivably cause intermittent cavitation, interrupting the current flow. The total force on the wall due to the pressure defect ks^{-2} in (22) is a logarithmically divergent integral. So is the integral of Maxwell's stress over the wall/fluid interface (i.e. the total reaction of the rest of the electric circuit on the currents in the fluid), and the integrated z-wise momentum flux of the fluid as $z \to \infty$. It is, however, easily verified that the 'Maxwell' force less the pressure-defect force equals the momentum flux and so no physical principle is offended.

A worthwhile extension of the present work would be to allow for a viscous boundary layer on the wall and the viscous spreading of the jet along the z axis. Note that, because the magnetic Prandtl number $\mu\sigma\nu$ is so small for real conducting liquids, it is realistic to take the magnetic Reynolds number K as small but the effective viscous Reynolds number $(J/\nu)(\mu/\rho)^{\frac{1}{2}}$ as large.

5. Perturbation of the current distribution

Provided K is small it is possible to calculate the departure from isotropy of the radial current flow through the origin due to electromagnetic induction, using the known flow and magnetic fields given by equations (17) and (14). Equation (9) then gives

$$\ddot{f} = \frac{K}{2\sqrt{2}\pi^2}\frac{(4\log 2-2)(2c+c^2)(1+c)^{-2}-\log(1+c)}{\{(4\log 2-1)c^2+c-(1+c)^2\log(1+c)\}^{\frac{1}{2}}}, \tag{23}$$

which is subject to the boundary conditions $f = \frac{1}{2}\pi$ at $c = 0$, and $f = 0$ at $c = 1$. When c is small, (23) becomes

$$\ddot{f} = \frac{K(2\log 2-\frac{5}{4})^{\frac{1}{2}}}{\pi^2}\left\{1-\frac{72\log 2-41}{48\log 2-30}c\ldots\right\}$$

and when c is near unity, with $\epsilon = 1-c$,

$$\ddot{f} = \frac{-K(\frac{3}{2}-2\log 2)^{\frac{1}{2}}}{4\pi^2\epsilon^{\frac{1}{2}}}\left\{1-\frac{3-2\log 2}{12-8\log 2}\epsilon\ldots\right\}.$$

Thus $\ddot{f}$ becomes infinite as $c \to 1$ but $\dot{f}$ and f are finite, and so j_r remains finite in this approximation, from (8). The solution of (23), found by numerical quadrature, is

$$f = (1-c)/2\pi + \delta f,$$

where $\delta f/K$ is presented in figure 3 as a function of c. Particular interest attaches to the values $c = 0{\cdot}281$ and $0{\cdot}896$, or $\theta = 73{\cdot}7^\circ$ and $26{\cdot}4^\circ$, at which $\delta\dot{f}$ changes sign and the perturbation current changes from being inwards to being outwards, as figure 4 shows. Thus the electromagnetic induction tends to shift the total current flow to the wall and the axis, which encourages the conjecture that at high values of K the current might be confined wholly to the wall and axis. It is not yet clear, however, how the changed current distribution alters the vorticity generation process. Zhigulev (1960) expressed the opinion that the current would be confined wholly to the axis at high K.

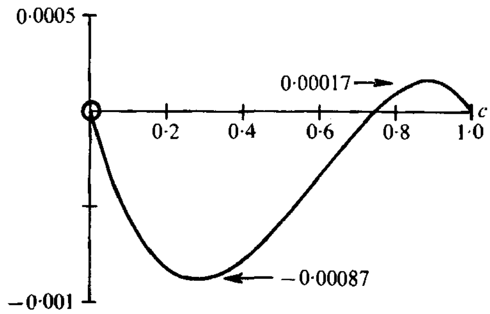

FIGURE 3

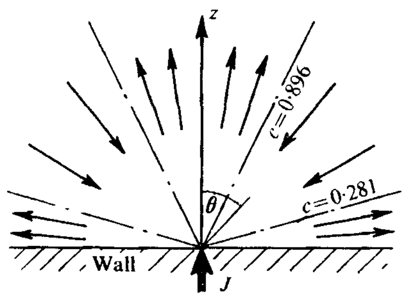

FIGURE 4. Perturbation currents at low K.

6. Concluding remarks

The analysis has revealed how the rotational $\mathbf{j} \times \mathbf{B}$ force field has the effect of sucking fluid in sideways and as Zhigulev remarked ejecting it as a jet normal to the interface. A version of this process also occurs in the more complicated situations that occur in practice, where the current often passes between two fluids, usually of very different and varying density, and where the current at the interface is not concentrated in a mathematical point and several length scales enter into the specification of the problem. It is evident that the simple mathematical procedures used here would be inadequate for such real problems, particularly when viscosity, surface tension, natural convection due to ohmic heating and heat transfer play an important role. Many important questions remain unanswered: for example, when the fluid is finite in extent, can the recirculating fluid attain such high velocities that the effective magnetic Reynolds number becomes large enough for the current flow pattern to be drastically altered by electromagnetic induction? The stability of the flow and of the jet in particular also await investigation.

It is worth noting that in some of the applications, the vigorous motion induced in the fluid is a vital or desirable feature of the application, as, for instance, in metal transfer in arc-welding or stirring of the melt in arc furnaces.

The flows discussed here belong to the general class of electrically-driven flows, one of the richer areas for investigation in MHD and of particular interest because it has no direct counterpart in ordinary fluid mechanics; the flows are not just ordinary motions modified by an imposed magnetic field. The present problem should be contrasted with those electrically-driven flows in which a magnetic field is imposed as well as imposed currents, as for instance in the work of Hunt & Malcolm (1968).

Another classification of the present study is to list it with those phenomena where the effect of the rotational $\mathbf{j} \times \mathbf{B}$ body force is wholly to *create* vorticity, in contrast to those more familiar cases in which its effect, some or all of the time, is to *suppress* vorticity.

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